Final Report

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Title of project	**Cavity quantum electrodynamics of systems from few atoms to controlled ensembles**
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Contents

1 Introduction

Cavity quantum electrodynamics (cavity QED, CQED) is devoted to studying the interaction of the electromagnetic radiation field with atoms in a space limited by boundaries. Beyond the initial motivation to observe substantial modification of the radiative properties of atoms due to the boundary conditions, current research is oriented mainly towards dynamical effects in a strongly coupled atom-radiation system. Whereas usual thermal light sources as well as lasers act on atoms merely as external fields, atoms in a cavity exert a significant back-action on the radiation mode. From a complementary point of view, while in ordinary optical setups the material component is represented by inert objects used for manipulating the light, the atoms, both the internal electronic and the centre-of-mass motional degrees of freedom, can evolve considerably under the influence of the cavity field. The essence of cavity QED, in a nutshell, is that the atomic and the electromagnetic components are genuine dynamical variables in a microscopic quantum system.

The prototype system of cavity QED is a single atomic dipole coupled to the electromagnetic radiation field in a tiny, high-finesse resonator (cavity) composed of mirrors with extremely high reflection coefficients. In this geometry, the strong enhancement of the interaction can be attributed to the "recycling" of photons by multiple reflections. The research objective of this OTKA project has been defined as to extend the basic ideas of CQED to other experimentally accessible systems where interesting consequences of the dynamical coupling may occur and can be used for various applications.

Owing to the development of neutral atom transport, cooling, and, in particular, cavity cooling methods during the last decade, atoms can be captured more than seconds in the tiny volume of Fabry–Pérot-type resonators[1]. The fine details of the dynamics on a frequency scale well below the recoil frequency[2] can be experimentally explored and thus become relevant for the theoretical description. Cavity QED, in this way, has been connected to the subject of manipulating the gross motion of neutral massive particles by optical means. In this research project, we focussed on the centre-of-mass motional degree of freedom of the atoms, and the internal electronic degrees of freedom were adiabatically eliminated.

The main direction of generalization concerned the material component: we addressed the problem of a quantum degenerate gas interacting with the cavity field. The timeliness of this work is due to the recent experimental achievements that Bose-Einstein condensates can be loaded into the resonator volume. Light forces on an ensemble of atoms are distinguished by the inherent role of a cavity-mediated interaction between remote atoms: the evolution cannot be interpreted in terms of independent single-atom processes as opposed to free space schemes. We applied mean-field and beyond mean-field models to describe this system, and launched the research on quantum many-body physics in the context of cavity QED (see Sect. 2).

The other direction of extending cavity QED was related to considering other mechanisms for confining the radiation field into a finite volume. We have studied the case of propagating fields in optical waveguides, where the field is confined only in the two

[1] S. Nussmann, K. Murr, M. Hijlkema, B. Weber, A. Kuhn, and G. Rempe, Nature Physics 1, 122 (2005)

[2] The recoil frequency is $\omega_R = \hbar k^2/m$, where k is the wavenumber of the optical field, m is the atomic mass; ^{87}Rb has, for example, $\omega_R \approx 46 \times 10^3\ \mathrm{s}^{-1}$. This is the characteristic frequency of the mechanical effects of the radiation field on atoms.

directions transverse to propagation. We transfered the concept of strong coupling and cavity cooling into the scattering geometry (Sect. 3.1). Another bunch of results was obtained from a one-dimensional scattering model which allows for describing the mechanical light forces exerted on atoms at a very general level, in arbitrary multipass interferometer, and allows for the description of collective motional modes of moving scatterers, too. This approach yielded a clarified connection between CQED and the general studies of opto-mechanics (Sect. 3.2) as well as lead to the extension of back-action effects into the physics of optical lattices (Sect. 3.3).

Finally, one participant of the project collaborated with the group of T. Kiss on studying quantum random walks (see Sect. 4). One of the first experimental realization of quantum random walk has been achieved in cavity QED[3], which justifies our interest in this subject of quantum information theory.

In the duration of the project, we have published altogether 21 papers, among which there are 6 Physical Review Letters, 8 Physical Review A, and 3 European Physical Journal D publications. We presented 7 invited conference talks, one contributed talk, and many posters. In the following we briefly present the main results of this project.

2 Bose-Einstein condensate in a cavity

Within a resonator, the mechanical effect of light on an ensemble of atoms relies on an intrinsic many-body system: there is a peculiar, indirect atom-atom interaction mediated by the cavity mode. This atom-atom coupling is of long range and gives rise to collective effects and, in particular, non-equilibrium phase transitions of an atomic cloud. One example is the spatial self-organization of atoms laser-illuminated externally from a direction perpendicular to the resonator axis. Above a critical pump strength, the homogeneous cloud undergoes a phase transition into one of the two possible, λ-periodically ordered configurations which is bound by the field scattered into the resonator (see Fig. 1). For a thermal gas of cold atoms, this transition has been predicted[4] in 2002, and experimentally demonstrated[5] in 2003. In this research project we studied the ultracold temperature limit[6] of this transition, i.e., the spatial self-organization of a Bose-Einstein condensate (BEC) in a high-finesse linear optical cavity. We showed that the steady-state of the pumped and lossy system exhibits an analogous transition at $T = 0$. The critical point was determined analytically from a mean-field theory. We calculated the lowest lying Bogoliubov excitations of the coupled BEC-cavity system (see Fig. 2) and the quantum depletion due to the atom-field coupling [2.5].

We considered the self-organization of atoms when the atomic motional degree of freedom is restricted to a two-mode space, i.e., to the one spanned by the homogeneous and the field-coupled sinusoidal modes. This subspace is sufficient to describe the phase transition, at the same time, the full quantum statistics can be taken into account. We showed that this beyond mean-field model corresponds to the spin-boson Dicke-model. The quantum phase transition of the Dicke-model from the normal to

[3] M. Karski, L. Förster, J. Choi, A. Steffen, W. Alt, D. Meschede, A. Widera, Quantum Walk in Position Space with Single Optically Trapped Atoms, Science 325, 174 (2009)

[4] P. Domokos and H. Ritsch, Phys. Rev. Lett. 89, 253003 (2002).

[5] A. T. Black, H. W. Chan, and V. Vuletić, Phys. Rev. Lett. 91, 203001 (2003).

[6] It is below the recoil temperature $k_B T < \hbar\omega_R$, i.e., below 100 nanoKelvin.

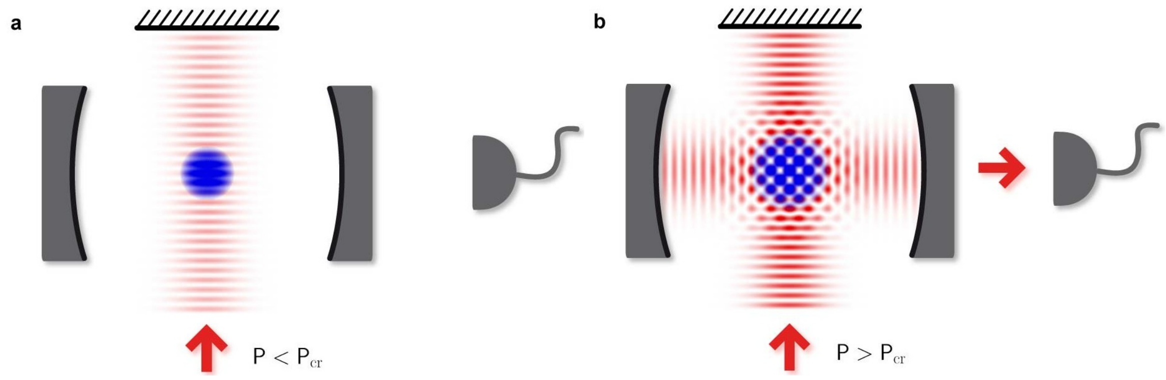

Figure 1: Schematic representation of the self-organization in 2D. A transverse pump field (plotted in red) illuminates directly the atoms, however, as long as the distribution (plotted in blue) along the cavity axis is quasi-homogeneous (a), there is no scattering into the cavity mode because of a destructive interference. Above a critical pump power (b), the atoms self-organize into a crystalline structure with wavelength period, constructively scatter light into the cavity mode (Bragg scattering from a crystal), and the pattern is bound by the scattered field interfering with the pump field.

the superradiant phase is thus analogous to the spatial self-organization of an ultracold atom cloud [2.1], that is, with the observation of this latter[7] a 50-year old research objective has been accomplished.

At zero temperature, atom-field entanglement plays a crucial role in the spatial reordering of the atoms from a homogeneous towards the two possible ordered states, where all atoms occupy either only even or only odd lattice sites. Concurrent with the buildup of atom-field entanglement, the homogeneous atomic cloud evolves immediately into the superposition of the two stable patterns entangled with opposite cavity field amplitudes. This possibility is absent in a factorized (classical) treatment of atoms and field and should be generic for spontaneous symmetry breaking in quantum phase transitions in optical potentials. This effect, as well as the dependence of the self-organization on quantum statistical properties (superfluid versus Mott-insulator initial atomic states) have been discussed in [2.8].

We have extensively studied the *open-system* quantum dynamics features of the coupled matter-light wave system, in which one of the components is subjected to dissipation. In the specific system, the dissipation is due to that photons can leak out through the highly reflective mirrors. The photon leakage can be interpreted also as a weak measurement process. We showed that quantum fluctuations of a cavity field couple into the motion of ultracold bosons and can be strongly amplified by a mechanism analogous to the Petermann excess noise factor in lasers with unstable cavities. For a Bose-Einstein condensate in a stable optical resonator, the excess noise effect amounts to a significant depletion on long timescales [2.4]. The fragility of the ground state due to photon measurement induced back action is calculated for the Dicke model quantum phase transition, too.

In order to gain more insight into the dynamics, we confined the excitation space of a Bose-Einstein condensate into a single relevant mode and derived a quantum master equation describing the coupled dynamics in a high-finesse optical cavity. This system is formally analogous to a broad class of opto-mechanical systems comprising vibrating mirrors and resonator modes coupled by radiation pressure. The presented equation accounts for the dissipative part of the dynamics due to the coupling

[7] K. Baumann, C. Guerlin, F. Brennecke, and T. Esslinger, Nature 464, 1301 (2010).

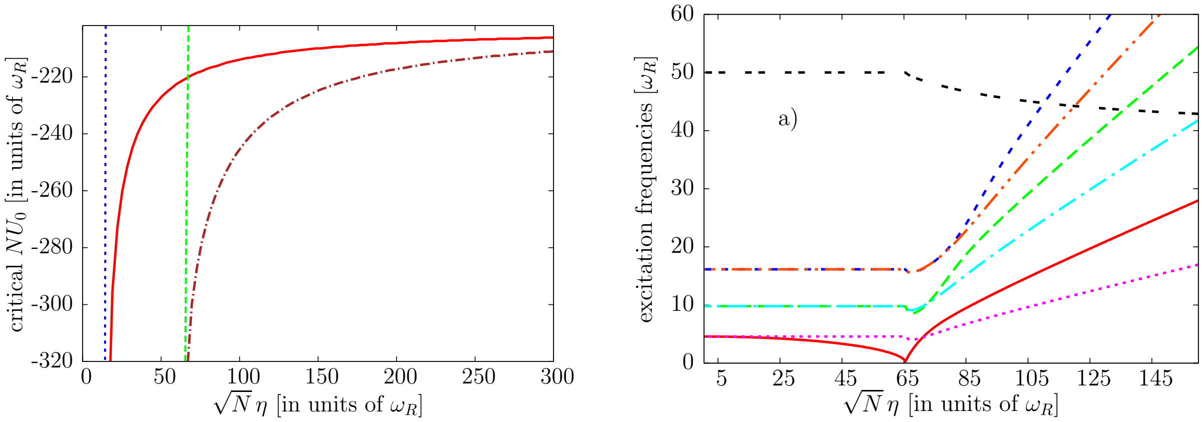

Figure 2: Left: Full phase diagram of the system, calculated form the mean-field theory, for two different atom-atom s-wave collision parameters $Ng_c = 0$ and $10\ \lambda\omega_R$. For $Ng_c = 0$ (collisions are suppressed), the dashed blue curve represents the phase boundary between homogeneous distribution and self-organized lattice. In the region between the dashed blue and the solid red line (these curves coalesce asymptotically for large NU_0) there is a self-organized lattice without defects. The solid red curve gives the critical NU_0 below which secondary trapping minima and defects occur in the self-organized lattice. Right shifted, the same phase boundary lines are drawn by dotted green and dashed-dotted brown for $Ng_c = 10\lambda\omega_R$. Right: Real part of the spectrum of the six lowest collective condensate excitations and the ones corresponding to the cavity field, showing the transition between the homogeneous gas (small η) and the almost harmonically trapped, self-organized gas. The critical point corresponds to the pump where the first excitation energy vanishes. Here $NU_0 = -100\omega_R$, so no defects can exist. Parameters are $\kappa = 200\omega_R$, $\Delta_C = -2\kappa$.

of a driven, lossy optical mode of a resonator. This allows for exploring the quantum limit of opto-mechanical systems in the presence of dissipation in a classically bistable regime. We found that the measurement-induced back-action noise impedes the observation of quantum tunneling and leads to a non-exponential dephasing of coherent matter wave oscillations [2.3].

We also studied the possibility of applying the mechanism of cavity cooling on molecules, in particular, on ensemble of molecules. Cooling of molecules via free-space dissipative scattering of photons is thought not to be practicable due to the inherently large number of Raman loss channels available to molecules and the prohibitive expense of building multiple repumping laser systems. The use of an optical cavity to enhance coherent Rayleigh scattering into a decaying cavity mode has been suggested as a potential method to mitigate Raman loss, thereby enabling the laser cooling of molecules to ultracold temperatures. We discuss the possibility of cavity-assisted laser cooling particles without closed transitions, identify conditions necessary to achieve efficient cooling, and suggest solutions given experimental constraints. Specifically, it is shown that cooperativities much greater than unity are required for cooling without loss, and that this could be achieved via the superradiant scattering associated with intracavity self-localization of the molecules. Particular emphasis is given to the polar hydroxyl radical (OH), cold samples of which are readily obtained from Stark deceleration [2.6].

[2.1] D. Nagy, G. Konya, G. Szirmai, and P. Domokos
Dicke-Model Phase Transition in the Quantum Motion of a Bose-Einstein Condensate in an Optical Cavity
Phys. Rev. Lett. 104, 130401 (2010)

[2.2] G. Szirmai, D. Nagy, and P. Domokos
Quantum noise of a Bose-Einstein condensate in an optical cavity, correlations, and entanglement
Phys. Rev. A 81, 043639 (2010)

[2.3] D. Nagy, P. Domokos, A. Vukics, and H. Ritsch
Nonlinear quantum dynamics of two BEC modes dispersively coupled by an optical cavity
Eur. Phys. J. D 55, 659–668 (2009)

[2.4] G. Szirmai, D. Nagy, P. Domokos
Excess Noise Depletion of a Bose-Einstein Condensate in an Optical Cavity
Phys. Rev. Lett. 102, 080401 (2009)

[2.5] D. Nagy, G. Szirmai, P. Domokos
Self-organization of a Bose-Einstein condensate in an optical cavity
Eur. Phys. J. D 48 (1), 127-137 (2008)

[2.6] B. L. Lev, A. Vukics, E. R. Hudson, B. C. Sawyer, P. Domokos, H. Ritsch, J. Ye
Prospects for the cavity-assisted laser cooling of molecules
Phys. Rev. A 77, 023402 (2008)

[2.7] D. Nagy, P. Domokos
Dipole-dipole instability of atom clouds in a far-detuned optical dipole trap
Phys. Rev. A 75, 053416 (2007)

[2.8] C. Maschler, H. Ritsch, A. Vukics, P. Domokos:
Entanglement assisted fast reordering of atoms in an optical lattice within a cavity at T=0
Optics Communications 273, 446-450 (2007)

3 Generalization of strong atom-field coupling for propagating fields

3.1 Optical waveguides

The single atom cavity QED produced a wealth of interesting phenomena which rely on the strong coupling between the atomic dipole and a radiation field mode confined in the small volume of a resonator. One would expect that such a strong coupling arises in the scattering geometry, too, where the light field is propagating and scatters off the atom. Naively, the geometric cross section of the impinging light beam must approach the resonant atomic cross section ($\sigma_A = \lambda^2/2\pi$) which is an easily accessible regime today with the advent of hollow-core photonic bandgap fibers. Inspired by the successful cavity cooling method, we revisited the theory of light forces on atoms in a waveguide.

Laser cooling of the thermal motion of atoms relies on the velocity dependence of the light force accompanying photon scattering processes. Since the Doppler effect can lead only to a fine tuning of the scattering cross section, some kind of resonant enhancement must be involved in the light-matter interaction. In the simplest case of "conventional methods", one makes use of atomic resonances driven by quasi-resonant laser sources. In an alternative approach, so-called *cavity cooling*, the atom is coupled to a

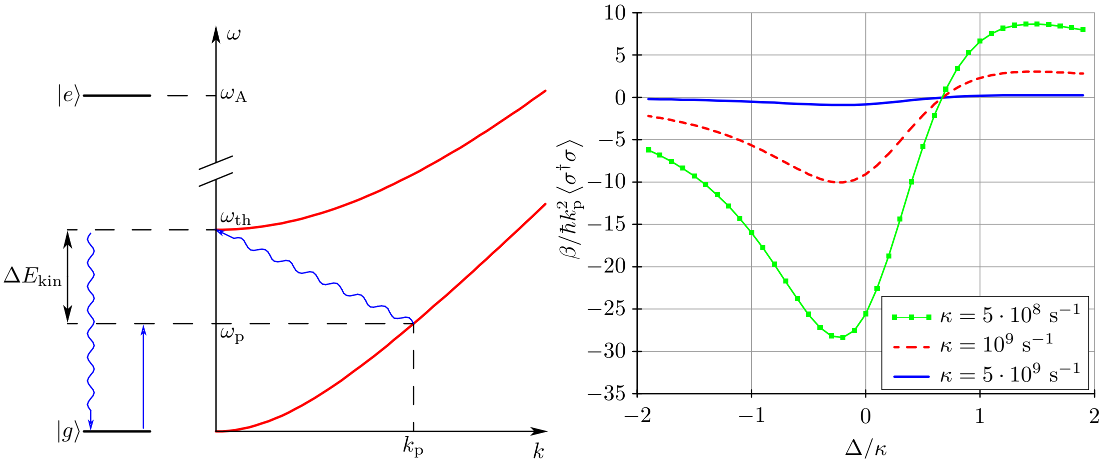

Figure 3: Left: The mechanism underlying the cooling in a waveguide described by using the generic dispersion relation of an optical waveguide (red curves in the left panel). Photons from the fundamental branch can be scattered into a different, transversely excited mode by the atom. The pump field frequency ω_p is close to, but below the threshold ω_{th} of the excited branch (figure is not to scale). The mode density is strongly enhanced at the threshold, so this scattering process has high probability. The photon frequency is converted up, thereby the atom loses kinetic energy. Right: The dependence of the linear friction coefficient on the detuning of the pump field from the threshold frequency, which exhibits a characteristic resonance curve as a function of the detuning $\omega_p - \omega_{th}$, κ is the waveguide loss parameter.

discrete mode of an optical resonator. Although the bare atomic transition frequencies can be very far from the external driving frequency, this latter can be close to a cavity eigenfrequency and thus to a polariton resonance of the coupled atom-field system. The virtue of this cooling scheme is that the atom behaves simply as a polarizable particle, i.e., its initial and final electronic states match.

We showed that optical cooling can be based on *geometric resonances* occurring in waveguides. Here the field is confined spatially only in two directions, and propagates along the third one. The spectrum of the radiation modes of a waveguide is continuous. Unlike in cavities, as the photons are not multiply recycled by reflections, the field cannot be described by a single or by a few degrees of freedom. Therefore, even the joint atom-field system does not manifest discrete resonances, being associated with an excited state of a given mode, which would be suitable for cooling. However, there is a discrete set of continuous branches, each of them belonging to a given spatial function in the transverse directions and to a given polarization. There are discontinuities, a signature of the geometric resonance, at the threshold frequency of the branches, that is, when a new transversally excited mode function fits in the waveguide cross section. The velocity-dependent force on atoms stems from the process when the pump field, injected in the fundamental branch of the waveguide, is scattered into one of the transversely excited branches of propagating modes. All fields involved are far detuned from any resonances of the atom, so the atom undergoes Rayleigh scattering. However, the scattering is inelastic, both the kinetic energy and the photon energy (frequency) can change in a prescribed manner. For a simple polarizable particle, linear friction force coefficient comparable to that of cavity cooling can be achieved. We presented and quantitaively described this waveguide cooling scheme using QED theory in [3.1.1].

[3.1.1] G. Szirmai, P. Domokos
Geometric resonance cooling of polarizable particles in an optical waveguide
Phys. Rev. Lett. 99, 213602-1–4 (2007)

3.2 Scattering model of opto-mechanics

The use of light forces to manipulate mechanical motion has been extended by now from the translational motion of single atoms to the motional modes of massive systems, such as the oscillations of a micro-mechanical mirror. The theoretical approach to describe the mechanical effect of light on the center-of-mass motion of atoms is completely distinct from the models dealing with vibrating optical resonators. In the first case, theories are based on the assumption that atoms are very weak scatterers in free space, negligibly perturbing the impinging bright laser beams. In the other case, the influence of the moving massive component on the radiation field is so strong that it is considered a (moving) boundary condition defining a single or a few modes of the field participating in the opto-mechanical coupling. This is clearly the case for a Fabry-Perot type resonator with one of its mirrors moving. We argue that these two cases can be dealt with as two extremes of a general system that can be described in a unified theoretical framework.

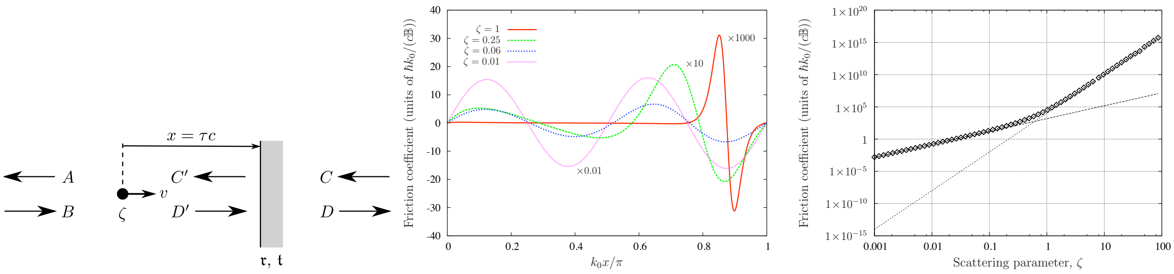

Figure 4: Left: The example considered: motion of a polarizable object, a general scatterer (mirror or atom), in front of a fixed, highly reflective mirror. Middle: The linear coefficient of the velocity-dependent friction force acting on the scatterer as a function of the position in a wavelength range (far from the mirror). Right: The dependence of the linear friction coefficient on the scatterer's polarizability at the optimised position within a wavelength range. The log-log scale covers a broad range including single atoms (small ζ on the left) and highly reflective mirrors (large ζ on the right).

We developed a scattering theory for opto-mechanically coupled systems [3.2.3], allowing for the efficient description of the motion of arbitrary combinations of atoms and mirrors interacting through the radiation field. The model is restricted to one-dimensional motion and small velocities, but multiple scattering to all orders is taken into account. The main building block is the beamsplitter transfer matrix, i.e., the *local relation* between light field amplitudes at the two sides of a scatterer. We calculated the radiation force acting on a moving scatterer up to linear order in the velocity. The model is completed by including the quantum fluctuations of the radiation force which stem from the quantized nature of the field. We determined the momentum diffusion coefficient corresponding to the minimum quantum noise level.

The theory was applied to describe the scheme of a Fabry-Perot resonator with one of its mirrors moving (Fig. 4). The friction force, as well as the diffusion, acting on the moving mirror is derived. In the limit of a small reflection coefficient, the same model provides for the description of the mechanical effect of light on an atom moving in front of a mirror. This result has been highlighted in Nature Photonics, Vol. 3, page 370 (2009), and in Physics, May 11, (2009).

In the framework of the one-dimensional scattering model we showed that the fields in multiple-pass interferometers, such as the Fabry-Pérot cavity, exhibit great sensitivity not only to the presence but also to the motion of any scattering object within the optical path [3.2.1]. We considered the general case of an interferometer comprising an arbitrary configuration of generic beam splitters and calculated the velocity-dependent radiation field and the light force exerted on a moving scatterer. We found that a simple configuration, in which the scatterer interacts with an optical resonator from which it is spatially separated, can enhance the optomechanical friction by several orders of magnitude.

[3.2.1] A. Xuereb, T. Freegarde, P. Horak, and P. Domokos
Optomechanical Cooling with Generalized Interferometers
Phys. Rev. Lett. 105, 013602 (2010)

[3.2.2] A. Xuereb, P. Domokos, P. Horak, and T. Freegarde
Scattering theory of multi-level atoms interacting with arbitrary radiation fields
Phys. Scr. T 140 (2010) 014010

[3.2.3] A. Xuereb, P. Domokos, J. K. Asbóth, P. Horak, T. Freegarde
Scattering theory of cooling and heating in optomechanical systems
Phys. Rev. A 79, 053810 (2009)

3.3 Collective motional effects in an optical lattice

Optical Lattices (OL) are generally produced using extremely far detuned lasers: detunings of about 10^7 times the atomic transition linewidth are not uncommon. On the one hand, this ensures that the dipole force dominates the scattering force, and the particles are only slightly heated by the light used to trap them. On the other hand, in this regime the particles do not affect the propagation of light very much, and thus optical back-action is avoided. The advantage is that this way light is a tool to produce an inert potential. However, in related systems, it is just the optical back-action that gives rise to interesting phenomena such as cavity-induced cooling of atoms, for example, or self-organization and other type of collective behaviours.

Let us notice that Bragg reflection from the optical lattice of atoms is a consequence of the back-action of individual atoms on the field. Therefore, it is justified to consider back-action effects provided the Bragg reflection is a measurable signature. In this spirit we started to study the interaction of large one-dimensional atomic ensembles with laser light, and to pave the way to doing many-body physics with radiative atom-atom coupling in free space.

We used a scattering model in which the trapped clouds of atoms are identified with single scattering centers, i.e., beam splitters in one dimension. We derived the optical force on such a beam splitter and generalize the standard "radiation pressure" and "dipole force". These latter can be considered as perturbative approximations to our exact result, in which the back-action of the scatterer on the optical field is neglected.

The exact result accounting for back-action can be interpreted by means of a simple physical picture, in terms of multiple reflection within the beam splitter.

The modification of the optical force, due to back action, on a single atom cloud is small. However, we have found that tuning a hitherto neglected parameter of an optical lattice its consequences can become striking. This parameter, the “asymmetry”, is the relative power of the pump beams constituting the trap. For symmetric OL’s — created by two counterpropagating beams with the same intensity — the back-action induced interaction leads to the reduction of the lattice constant d mentioned above, and we predict that it also causes the center-of-mass oscillations to soften. In asymmetric lattices, the reduction of the lattice constant d is enhanced (in some cases by several orders of magnitude), and arises even for blue detuning, where the first intuition would suggest that d increases. However, this equilibrium phenomenon is masked by dynamic effects. The interaction induced by back-action leads to a dynamic instability of

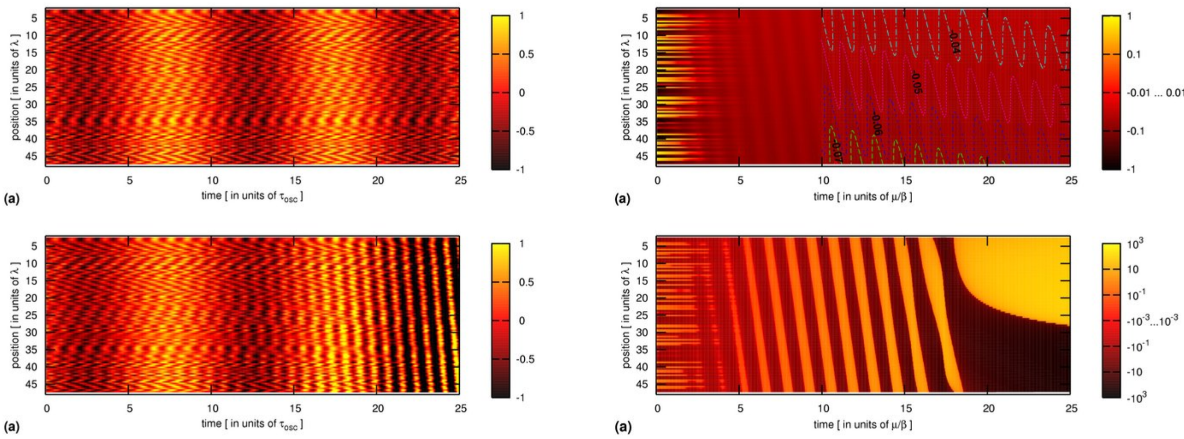

Figure 5: Numerical simulation of the dynamics of an optical lattice of $N = 100$ clouds with polarizability $\zeta = 0.1$ each (the cloud is an ensemble of atoms located t the same trapping site in the lattice, and the collective polarizability is denoted by ζ), after initial excitation (random displacement from the equilibrium with maximum magnitude $\xi = 5 \cdot 10^{-4}\lambda$ and random velocity of maximum magnitude $\omega_{\rm osc}\xi$, where $\omega_{\rm osc}$ is the frequency in the individual trapping wells). The clouds lie along the vertical axis (in wavelength unit) and their displacement from the respective equilibrium position is represented by color coding as a function of time (horizontal axis, in units of the oscillation time $\tau_{\rm osc} = 2\pi/\omega_{\rm osc}$). Left, upper panel: symmetric pumping, $I_0 = I_1$; bottom (left): asymmetric pumping $I_1 = 1.47 I_0$; no friction is assumed ($\mu = 0$). The softened centre-of-mass mode can be observed above (the oscillation frequency is $\sqrt{N}$ times of the individual oscillation frequencies). Below: the formation of density waves, which are collective excitations, can be seen, for example, from the black lines in skew angle correspond to a propagating node along the chain formed at about 15 $\tau_{\rm osc}$. Right: asymmetric pumping in the overdamped limit by assuming large friction ($\mu \to \infty$). Upper panel (right): $I_1 = 1.87 I_0$, bottom: $I_1 = 1.88 I_0$. While the initial random excitations as well as the density waves therefore die out in the upper panel, a slight change of the asymmetricity leads to density wave amplification and to instability of the optical lattice. This reveals the criticality in the system.

optical lattices, which happens already at moderate pump asymmetries. Small lattice fluctuations give rise to an exponentially increasing density wave propagating along the direction of the weaker beam, which ultimately leads to a destruction of the lattice: the particles will all be pushed away by the stronger beam. Viscous friction can prevent this instability, but even with an arbitrary amount of friction (overdamped limit) there is a critical asymmetry beyond which the OL becomes unstable. Curiously, the propagating density waves arise in the overdamped limit as well, in spite of the fact that the dynamics is first-order. We study these effects numerically and analytically, providing an analytical characterization of the density waves and closed formulas for the critical asymmetries.

We solved the self-consistent, coupled equations of motion for trapped atoms and the field of a one-dimensional optical lattice. The solution shows explicitly that the dynamics cannot be described in terms of an "optical potential", an approach often encountered in the literature. We have discussed the shortcomings of the potential energy approach, and shown different derivations of the optical force, in this 1-dimensional model. In two recent articles[8,9], Meiser and Meystre describe the coupled dynamics of a dense gas of atoms and an optical cavity pumped by a laser field. They make two important simplifying assumptions: (i) *the gas of atoms forms a regular lattice and can be replaced by a fictitious mirror*, and (ii) *the atoms strive to minimize the dipole potential*. We show that the two assumptions are inconsistent: the configuration of atoms minimizing the dipole potential is not a perfect lattice. Assumption (ii) is erroneous, as in the strong coupling regime the dipole force does not arise from the dipole potential. The real steady state, where the dipole forces vanish, is indeed a regular lattice. Furthermore, the bistability predicted does not occur in this system.

[3.3.1] J. K. Asbóth, H. Ritsch, P. Domokos
Optomechanical coupling in a one-dimensional optical lattice
Phys. Rev. A 77, 063424 (2008)

[3.3.2] J. K. Asbóth, P. Domokos
Comment on "Coupled dynamics of atoms and radiation–pressure–driven interferometers"
Phys. Rev. A 76, 057801-1–4, (2007)

[3.3.3] J. K. Asbóth, H. Ritsch, P. Domokos
Collective excitations and instability of an optical lattice due to unbalanced pumping
Phys. Rev. Lett. 98, 203008 (2007)

4 Quantum random walk

We analyzed the realization of a quantum-walk search algorithm in a passive, linear optical network [4.1]. The specific model enables us to consider the effect of realistic sources of noise and losses on the search efficiency. Photon loss uniform in all directions is shown to lead to the rescaling of search time. Deviation from directional uniformity leads to the enhancement of the search efficiency compared to uniform loss with the same average. In certain cases even increasing loss in some of the directions can improve search efficiency. We showed that while we approach the classical limit of the general search algorithm by introducing random phase fluctuations, its utility for

[8] D. Meiser and P. Meystre, Phys. Rev. A **73**, 033417 (2006).
[9] D. Meiser and P. Meystre, Phys. Rev. A **74**, 065801 (2006).

searching is lost. Using numerical methods, we found that for static phase errors the averaged search efficiency displays a damped oscillatory behavior that asymptotically tends to a nonzero value.

The SKW quantum random-walk search algorithm[10] is known to require $\mathcal{O}(\sqrt{N})$ number of oracle queries to find the marked element, where N is the size of the search space. The overall time complexity of the SKW algorithm differs from the best achievable on a quantum computer only by a constant factor. We presented improvements to the SKW algorithm which yield a significant increase in success probability [4.2], and an improvement on query complexity such that the theoretical limit of a search algorithm succeeding with probability close to one is reached. We pointed out which improvement can be applied if there is more than one marked element to find.

We presented the first robust implementation of a coined quantum walk over five steps using only passive optical elements [4.3]. By employing a fiber network loop we keep the amount of required resources constant as the walker's position Hilbert space is increased. We observed a non-Gaussian distribution of the walker's final position, thus characterizing a faster spread of the photon wave packet in comparison to the classical random walk. The walk is realized for many different coin settings and initial states, opening the way for the implementation of a quantum-walk-based search algorithm.

[4.1] A. Gábris, T. Kiss, I. Jex
Scattering quantum random-walk search with errors
Phys. Rev. A 76, 062315, 2007

[4.2] V. Potocek, A. Gábris, T. Kiss, I. Jex
Optimized quantum random-walk search algorithms on the hypercube
Phys. Rev. A 79, 012325 (2009)

[4.3] A. Schreiber, K. N. Cassemiro, V. Potoçek, A. Gábris, P. J. Mosley, E. Andersson, I. Jex, Ch. Silberhorn
Photons Walking the Line: A Quantum Walk with Adjustable Coin Operations
Phys. Rev. Lett. 104, 050502 (2010)

5 Summary

This project has led to the extension of the basic cavity QED model in various directions, connecting it to other very active research fields. First of all, the strong atom-field coupling has been incorporated to the physics of Bose-Einstein condensates. The composite, hybrid system of coupled matter and light waves has been explored, first, by mean field theory and linearized fluctuation analysis. We developed effective models accounting for quantum statistical features, too, even for dynamical effects. The non-equilibrium phase transition of spatial self-organization has been analysed in detail. Experimental observations in the ETH group verified some of our predictions.

We established a link between cavity QED and the standard opto-mechanical model. This latter is a hot topic with the promise of observing quantum effects in the kinematics of massive, mesoscopic mirrors. We described them in a uniform framework which includes the cavity QED and the standard opto-mechanical models as the two extremes of the same scattering model. This scattering approach proved to be useful

[10] Shenvi, Kempe, and Whaley, Phys. Rev. A 67, 052307 (2003)

also in studying collective atomic effects in free space. We went beyond the standard picture of optical lattices: we found corrections to the optical dipole potential due to back-action on the field and showed that many-body excitations in the form of density waves can be excited in an optical lattice.

This project grant allowed for the formation and stabilization of a small theoretical group working in the rapidly progressing field of ultracold atoms and opto-mechanics. The specific feature of our approach connecting these two, otherwise quite distinct fields is based on cavity quantum electrodynamics. In this way we could contribute by original results to timely and fashionable subjects, and our publications have already received a remarkable number of citations. In the course of the project, three postdoctoral fellows have been employed on the research grant. First, it was Gergely Szirmai, who joined the group with a solid many-body physics background. The next employee was David Nagy, after having completed his PhD in the group in the duration of the project. Finally, Orsolya Kálmán joined the project and got involved in the physics of Bose-Einstein condensates. They all could acquire significant experience in cavity QED, and more generally, in quantum optics, and thus they will be able to represent these research fields in our institute.